

Certificate in *Mathematical Methods*

Distance learning

“The motivating factor for designing the CM² has been the overwhelming interest developed by past 7city/Wilmott delegates for acquiring greater knowledge of the classical branches of mathematical methods, which have a wide range of 'real-world' applications.”

Certificate in Mathematical Methods

The Certificate in Mathematical Methods (CM²) is an intensive program covering a variety of mathematical methods, with special focus on those which are applicable to real-world problems. Through 39 recorded lectures, delegates will learn subjects that are normally covered in the first two years of a university mathematics degree. These sessions will be accompanied by problem sheets, designed to develop confidence and hence extend the course participants' understanding of the material covered in the lecture.

The impact of mathematics in the real world has never been greater. In addition to the classical branches of science and engineering, we are now seeing the application of mathematics to medicine, economics and finance, political and social science as well established sections of applied mathematics. The general view is that mathematics has "hijacked" many of the other well known non-mathematical disciplines. Naturally, the demand for good mathematicians globally is at an all-time high, while the availability of such trained professionals who are able to apply these skills in the workplace continues to lag. Simply put, demand exceeds supply.

The CM² has been designed with all of this in mind, but the most important motivating factor has been the overwhelming interest developed by CQF alumni for acquiring even greater mathematical knowledge and with a strongly applied flavour.

The course syllabus will include the following topics:

- Numerical Analysis
- Functions of a Complex Variable
- Linear and Nonlinear Differential Equations
- Partial Differential Equations
- Integral Equations
- Perturbation Methods
- Transform Methods

Your Questions Answered

Is this course suitable for me?

If you enjoyed the level of mathematics covered in the CQF, and that has developed your interest in obtaining further knowledge in those topics at an advanced level, then the course will be particularly suitable. During the course of the program we will concentrate on the actual mathematics in greater detail. Once instruction in these methods is obtained, delegates will have increased confidence in applying them to model a wide class of real-world situations.

Will there be any exams and if so, what form will they take?

The course is split into two terms. The first term consists of 20 recorded lectures and the second term consists of 19 recorded lectures. Each term will be assessed with an examination paper which delegates must pass (with 60% or over) in order to gain the certificate. Exams will be distributed four times a year on 27th March, 26th June,

25th September and 11th December 2009. Delegates are required to contact the CMM Programme Manager, Claire Davies on c.davies@7city.com to confirm their preferred date for their term 1 and term 2 exams at least two weeks prior to the exam date. The style and standard of questions will be typical of university mathematics degree exams. The examinations will be open book, take home exams. Candidates will receive the papers on a Friday and must submit the completed exam 10 days later.

What knowledge will I have by the end of the course?

The topics covered are all applied mathematics and mathematical methods. The aim of the course is to provide delegates with a solid grounding in the theory and use of mathematical techniques which can be applied to a wide range of problems.

Course Syllabus

Advanced Calculus

- **Complex numbers:** Loci in the Argand diagram, roots of unity (De Moivre's theorem) and applications.
- **Vector algebra:** Scalar and vector products including triple products. Cartesian components and direction cosines. Applications to 3-dimensional geometry: lines and planes.
- **Matrix algebra:** Matrix arithmetic; special matrices, determinants, inverses.
- **Ordinary differential equations:** First order (Bernoulli; exact; homogeneous). Linear equations of second and higher order (constant and variable coefficient problems). Theory of linear equations and Wronskians. D-operator method.
- **Infinite series:** Tests for convergence; power series - radius of convergence, absolute convergence.
- **Functions:** Gamma and Beta.
- **Calculus for several variables:** Partial differentiation (chain rules); Taylor series; tests for extrema; double and triple integration; integration over plane areas and volumes; change of variables; Jacobians.
- **Vector calculus:** gradient, divergence, curl, line integrals.

Linear Algebra

- **Linear equations:** Matrix formulation and elementary operations. Reduction to row echelon form.
- **Vector spaces:** Subspaces of \mathbb{R} and \mathbb{Q} ; Linear independence; Basis and dimension.
- **Linear mappings:** Rank and nullity.
- **Eigenvalues and eigenvectors:** Characteristic equation; diagonalisation of matrices, applications; Cayley-Hamilton Theorem
- **Gram-Schmidt process:** Orthogonal diagonalisation of real symmetric matrices.

Introduction to Probability

- **Introduction:** Axioms of a probability space (sample space and events), simple laws, independence, conditional probability and Bayes Theorem.
- **Random Variables:** Discrete random variables; standard distributions. Expectations.
- **Continuous random variables:** Densities, standard distributions.
- **Multivariate Random Variables:** Bivariate RV's. Joint Distribution Functions. Discrete and continuous random variables. Covariance and correlation. Special distributions.

Numerical Analysis I

- **Errors:** Sources and controls
- **Roots of equations:** Bisection, Newton-Raphson, False position, Secant method
- **Interpolation:** Lagrange and Inverse
- **Numerical Linear Algebra:** Direct methods (Gaussian elimination, LU decomposition, Cholesky); Iterative Techniques (Jacobi, Gauss Seidel, SOR); Eigenvalues (Power method).
- **Integration:** Trapezoidal and Simpson's rules; Gaussian quadrature.
- **Differential equations:** Euler and Runge-Kutta methods.

Complex Variables

- **Basic Properties:** Functions, limits, continuity.
- **Elementary functions:** e^z , $\sin z$, $\cos z$, $\log z$, z^a
- **Complex Differentiation:** Holomorphic functions, singular points. Cauchy-Riemann equations.
- **Complex Integration:** Curves, winding number, integration, Cauchy's theorem, Cauchy's integral formula.
- **Infinite Series:** Power series. Taylor's theorem, Laurent's theorem. Classification of singularities – poles, removable/essential singularities, branch points.
- **The Theory of Residues:** Calculation of residues, residue theorem, application to definite integrals (proper and improper), Jordan's Lemma.
- **Zeros of polynomials:** Fundamental theorem of algebra. Rouché's Theorem.
- **Conformal Mappings:** Transformations in the complex plane (translation, rotation, stretching, inversion). Riemann's Mapping Theorem.

Differential Equations

- **Fourier series:** Periodic functions, approximating series, formulae for coefficients.
- **Variation of parameters:** Linear first order and higher orders; systems of linear equations.
- **Linear ordinary differential equations:** Power series solutions - ordinary and regular/irregular singular point; Frobenius method. Solutions of complex integral form with applications to Airy equations.
- **Non-linear ordinary differential equations:** First order systems in two variables and linearization; singular points and classification; phase plane analysis.

Mathematical Methods

- **Elliptic Equations and related methods:** Laplace's equation in cylindrical coordinates and its solution by the separation of variables. Legendre's equation and its series solution. Legendre Polynomials, P_n , their orthogonality, and their generating function. Applications of the generating function.
- **Mathematics of Hyperbolic Equations:** The wave equation in cylindrical coordinates and its solution by separation of variables. Bessel's equation. The general Frobenius method and its application to Bessel's equation. J_n , Y_0 . The generating function for J_n and its application.

Advanced Mathematical Methods

- **Asymptotic expansions of integrals:** Laplace's method, Watson's lemma; methods of stationary phase and steepest descent.
- **Non-linear ordinary differential equations:** Second order equations with continuous or discontinuous damping terms. Free and forced oscillations. Limit cycles.
- **Integral Equations & Boundary-Value Problems:** Fredholm and Volterra equations; Sturm-Liouville Theory.
- **Perturbation Methods:** Matched asymptotic expansions and uses in solving ODE's and PDE's; boundary-layers.

Transform Methods

- **Laplace and Fourier transforms:** Their definition and inversion using tables in conjunction with the shift and dilation theorems and the convolution theorem. Inversions using complex integration, including integration around branch cuts.
 - **Applications:** Their use in the solutions of ODEs and PDEs.
-

Numerical Analysis II - Finite Difference Methods

- **Parabolic equations:** One, two and three dimensions. Explicit, implicit and Crank-Nicolson schemes. Stability and convergence criteria; error analysis. ADI method. Upwind differencing.
- **Hyperbolic equations:** One space dimension; Method of Characteristics; Lax-Wendroff scheme, error analysis.
- **Elliptic Equations:** Linear two dimensional problem.



Course Tutors

Riaz Ahmad

Dr Riaz Ahmad received advanced degrees in mathematics from University College London and Imperial College London. He has held academic positions at Imperial College, Lahore University of Management Sciences (LUMS), Pakistan and more recently Oxford University (Mathematical Institute), where he was also assistant academic director of the university's M.Sc. Mathematical Finance Program. Riaz is full-time director at 7city for all mathematical and computational finance based courses. In addition, he oversees 7city's Quantitative Finance series and consults on mathematical finance issues to City institutions. His research interests are in theoretical and numerical/computational methods for derivative pricing and Islamic Banking. Riaz is a visiting faculty member at LUMS where he teaches executive education courses in derivatives to finance practitioners (as well as on the MBA).



Paul Wilmott

Dr Paul Wilmott is internationally renowned as a leading expert on quantitative finance. His research work is extensive, with over 100 articles in leading mathematical and finance journals, as well as several internationally acclaimed books on mathematical modelling and derivatives, including the best-selling *Derivatives* and *Paul Wilmott on Quantitative Finance*, both published by John Wiley & Sons. Paul obtained mathematics degrees at the bachelor and doctorate level from Oxford University, where he also founded and directed the Mathematical Finance Group. Paul has extensive consulting experience in quantitative finance with leading US and European financial institutions. He has founded a financial training company and a university degree course. Paul has lectured at all levels, to students and to practitioners. Paul was Partner with responsibility for volatility forecasting and risk management at a statistical arbitrage hedge fund.



Duration: 39 2½ hour recorded sessions

Cost: £9,950 + VAT/ \$16,999

(discounted price of £2,950 + VAT/ \$5,995 for delegates enrolled on the CQF programme)

For further details on the Certificate in Mathematical Methods, or any other quantitative finance programmes, please contact:

7city Learning

4 Chiswell Street
London
EC1Y 4UP
United Kingdom

t: +44 (0)20 7496 8600 **e:** cqf@7city.com **w:** www.7city.com/quants

Other Quantitative Finance courses include:

- Certificate in Quantitative Finance
 - Including Certificate in C++ for Quantitative Finance
- Maths Primer